Introduction to Fuzzy Logic

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Fuzzy Set Introduction

Introductory Concepts of Fuzzy Logic

Crisp Sets

- Well defined sets with boundaries

Fuzzy Sets

- Loosely defined set without boundaries

Fuzzy and crisp sets

Fuzzy Logic Introduction

Introductory Concepts of Fuzzy Logic

Crisp Logic

- Crisp logic is a **two-value logic** representing two possible solution states, often represented by yes/no, 0/1, black/white, or true/false.

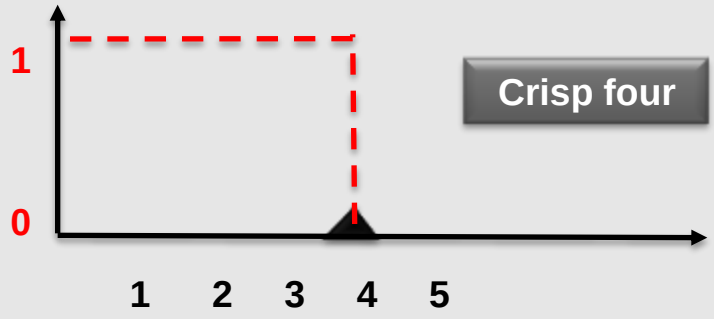
Fuzzy Logic

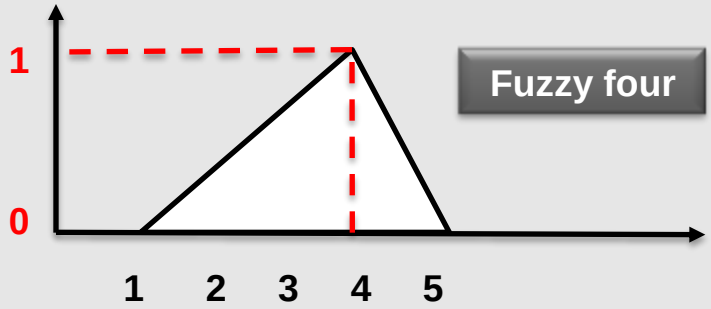
- Fuzzy logic is a **multivalued logic** introduced by Zadeh (1965) that allows intermediate values to be defined between the two aforementioned conventional evaluations.

Fuzzy and crisp concepts

Fuzzy Membership Functions

Crisp and Fuzzy Number



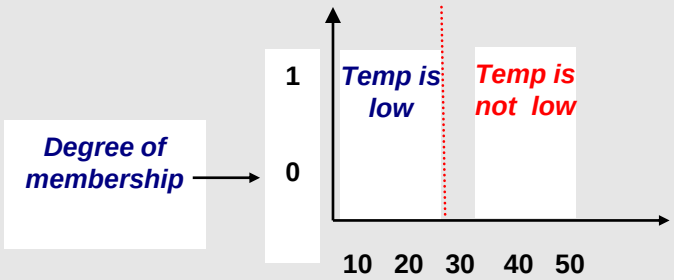


Humans **routinely and subconsciously** place things into classes whose meaning and significance are well understood but whose **boundaries are not well defined**.

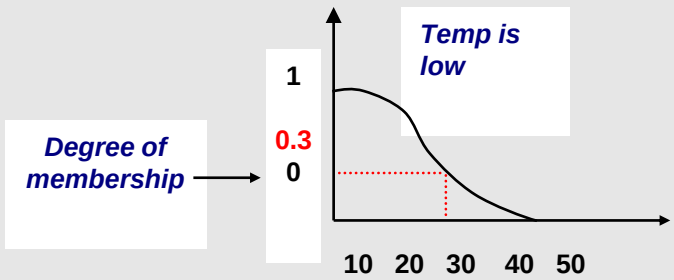
Hot season, large car, young boy and rich people are the examples for the same.

Examples of Fuzzy Membership Functions

Temparature



(A) Crisp set

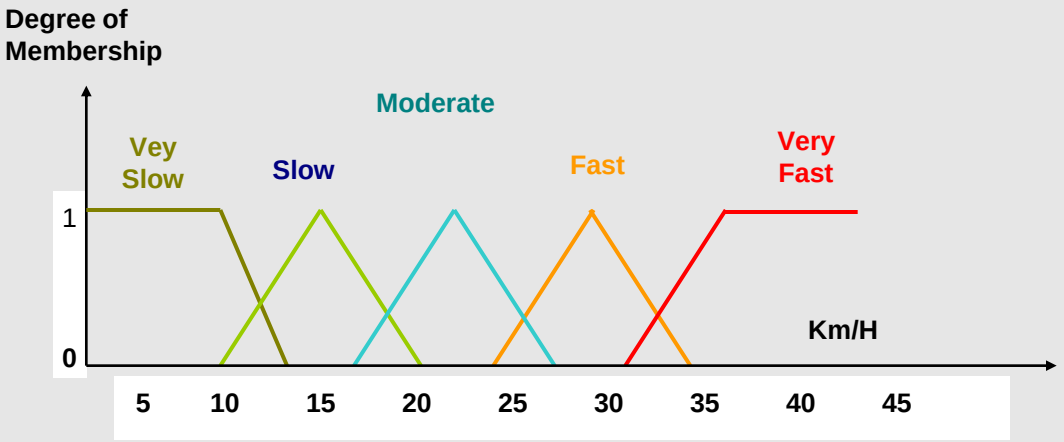


(B) Fuzzy set

Crisp and fuzzy sets for low temperature

Examples of Fuzzy Membership Functions

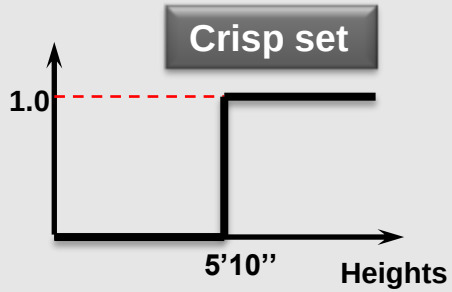
Speed

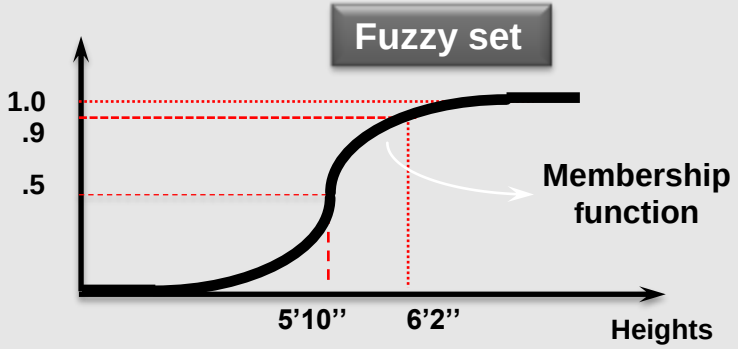


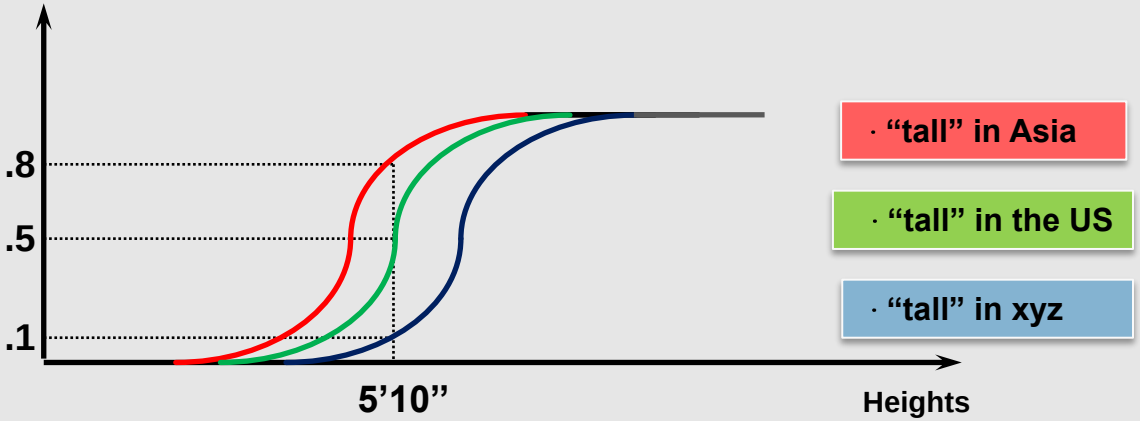
Fuzzy membership functions for typical air conditioner control

Examples of Fuzzy Membership Functions

Height







Examples of Fuzzy Membership Functions

Temparature

Hot

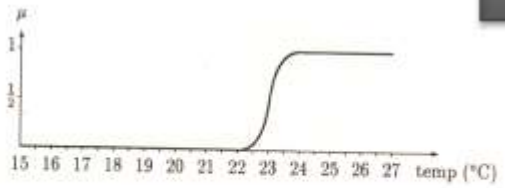


Fig. 2.2. The fuzzy notion hot.

Cold

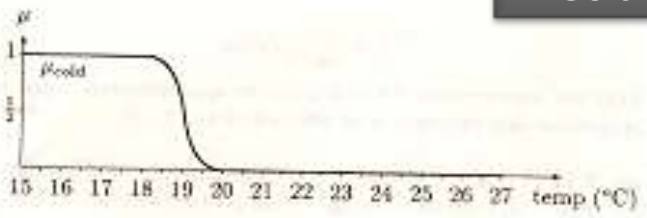
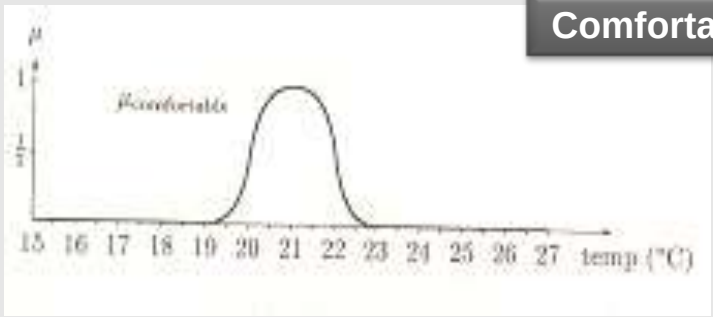
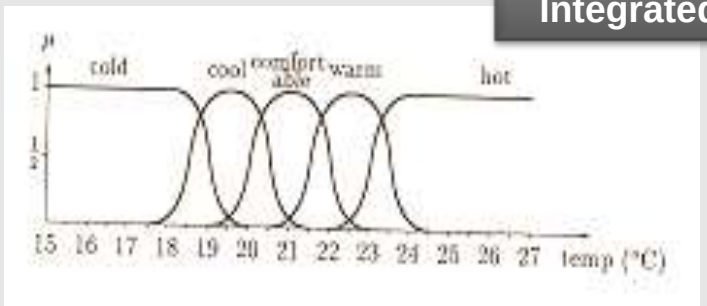


Fig. 2.3. The fuzzy notion cold.

Comfortable



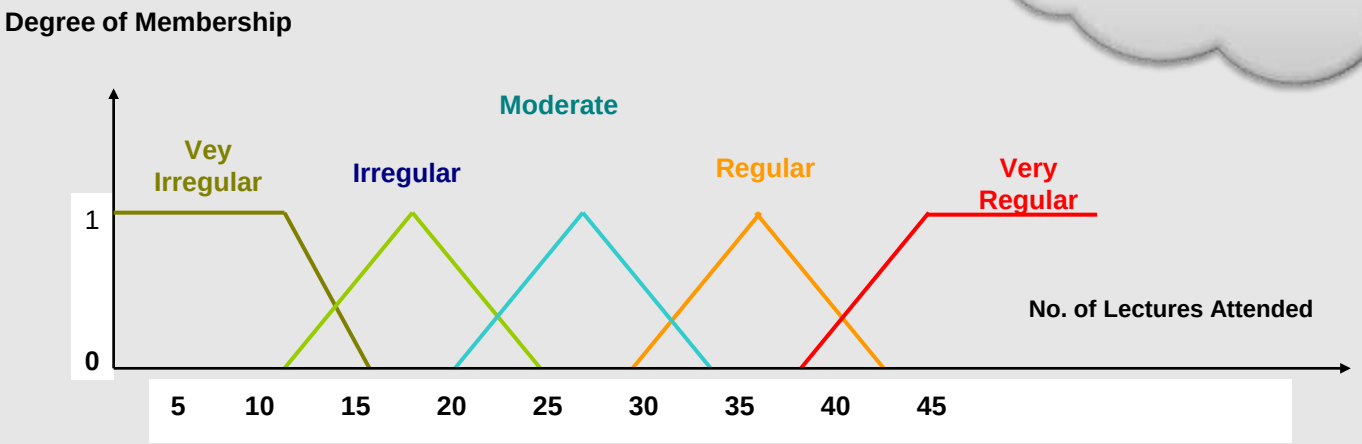
Integrated



Examples of Fuzzy Membership Functions

*Try to design fuzzy membership function for **strict teacher***

Irregular Student



Fuzzy membership functions for irregular student

Fuzzification and Defuzzification

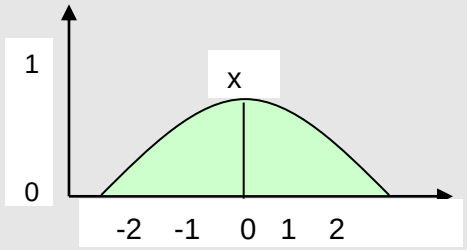
Fuzzyfication

- The process of transforming **crisp input values into linguistic values** is called “fuzzification.”

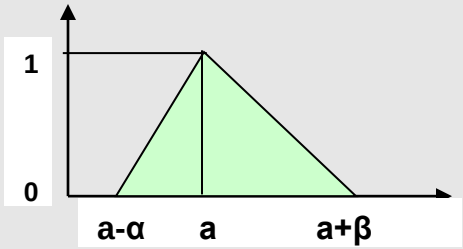
Defuzzification

- “Defuzzification ”converts the **fuzzy value into a crisp value**. It is the process of producing a quantifiable result from the fuzzy linguistic variable used.

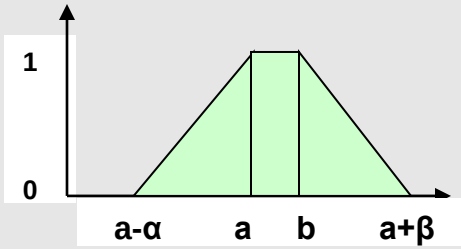
Types of Fuzzy Membership Functions



Quasi fuzzy membership function

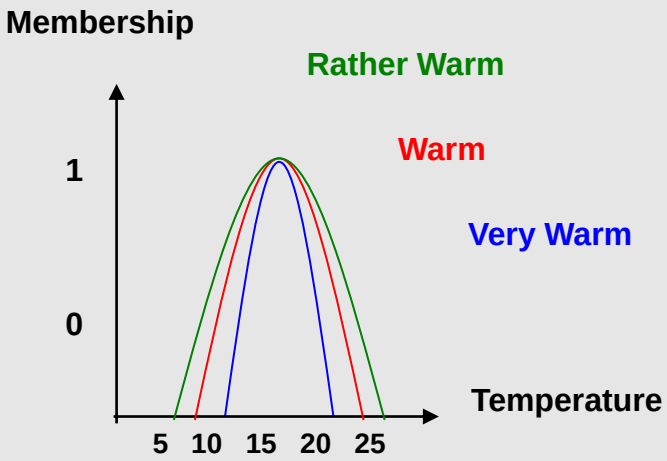


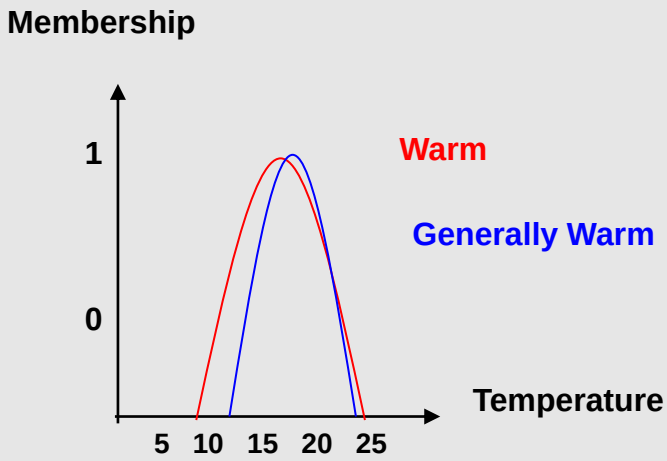
Triangular fuzzy membership function



Trapezoidal fuzzy membership function

Fuzzy Headges

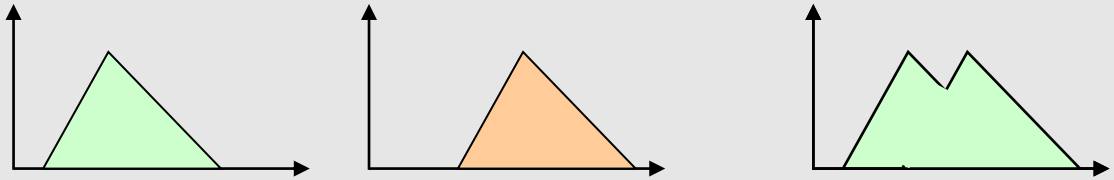




Linguistic hedges

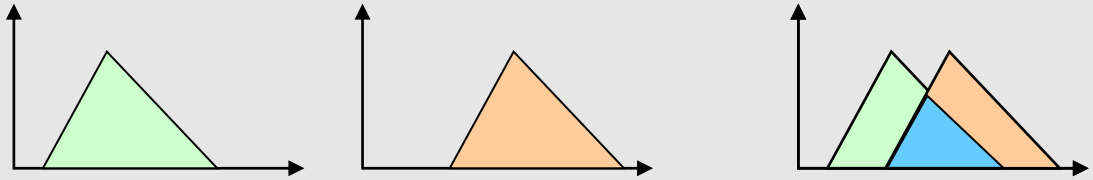
Operations on Fuzzy Sets

Fuzzy Union



Union of fuzzy sets

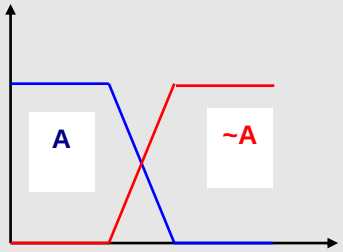
Fuzzy Intesection



Intersection of fuzzy sets

Operations on Fuzzy Sets

Fuzzy Negation



Negation of fuzzy sets

Fuzzy Membership Fucntion :Practical Example

Book Affordability Example

- Consider set of books B. The graded membership A of a book to determine affordability of a given book is defined as follows:
- A=0, if price of the book is more than 4999 Rs.
- A= 1-{price/500 } otherwise.
- Consider following information about the book and prices and calculate affordability of each book.

Book	Price (Rs.)	Affordability (A)
Book1	5000	= 0
Book2	400	= 1- 400/500 ➔ 0.2
Book3	300	?
Book4	150	?
Book5	400	?
Book6	100	?

Fuzzy Membership Fucntion :Practical Example

Book Quality Example

- Consider another graded membership **Q** referred to **quality of a book** given as :

Book	Quality
Book1	1
Book2	0.5
Book3	0.8
Book4	0.6
Book5	0.2
Book6	0.3

- Consider a fuzzy relationship **R** that determines the books to be purchased depending on good quality and affordable price.
- **Calculate the R values** for each book in the tables mentioned above**.**
- **Find out the most affordable as well as good quality book (simultaneously) as recommendation.**

Fuzzy Membership Fucntion :Practical Example

Solution

Book	Price	Affordability	Quality	Recommendation (R)
1	4999	0	1	0
2	400	0.2	0.5	0.2
3	300	0.4	0.8	0.4
4	**150**	**0.7**	**0.6**	**0.6**
5	400	0.2	0.2	0.2
6	100	0.8	0.3	0.3

The affordability **A**, which you have calculated

The quality **Q**, which is given

Crisp and Fuzzy Reltions

Crisp Relation

- Consider the set of machines, ***M***, and the set of people, ***P***, defined as follows:
- ***M*** = { set of all machines in a domain}
  - e.g., ***M*** = { *m*1, *m*2, *m*3, …, *mn*} where n is a finite small number

- ***P*** = {set of people}
  - e.g., *P* = { *p*1, *p*2, *p*3, …, *pk*} where k is a finite small number

- If the machines of set ***M*** are used by the people of set ***P***, the relationship, ***R***, can be defined as a relationship of ***M*P*** and identified with the "***used by***" phrase.
- Here, ***R*** is a subset of ***M*P*** and denoted as ***R (subset of) M*P*.** The individual relationship can be presented as follows:
  - (*p*1, *m*1), (*p*2, *m*2), (*p*3, *m*3),….

- Such relationships are crisp in nature and easy to handle mathematically.

Crisp and Fuzzy Relations

Fuzzy Relation

- Relationships like "**comfort of a person while working with a machine**" is really a fuzzy relationship and comparatively difficult to handle with crisp logic.
- The fuzzy relationship "generally comfortable" can be defined as follows:

	$m1$	$m2$	$m3$
$p1$	1.0	0.4	0.7
$p2$	0.8	1.0	0.6
$p3$	0.7	0.6	1.0

Crisp and Fuzzy Relations

Fuzzy Relation

- Consider U = V = {1,2,3}. The relationship of U*V, defined as "approximately equal," is a binary fuzzy relationship given by 1/(1,1), 1/(2,2), 1/(3,3), 0.8/(1,2), 0.8/(2,3), 0.8/(2,1), 0.4/(1,3), and 0.4/(3,1).
- **The following table presents the situation in matrix form.**

X/Y	1	2	3
1	1.0	0.8	0.4
2	0.8	1.0	0.8
3	0.4	0.8	1.0

- **The membership functions can be defined as:**

$$\mu R(x) = \begin{cases} 1.0 & \text{for } x = y \\ 0.8 & \text{for } |x-y| = 1 \\ 0.4 & \text{for } |x-y| = 2 \end{cases}$$

Crisp and Fuzzy Relations

Fuzzy Relation

- The intersection and union operators are also applicable to fuzzy relationships. They are defined as follows:

$$\mu R \cap S(x,y) = \min(\ \mu R(x,y),\ \mu S(x,y)\ )$$
$$\mu R \cup S(x,y) = \max((\ \mu R(x,y),\ \mu S(x,y)\ )$$

Examples: What is R∩S ?:

Consider U = V = {1,2,3}. Relationship R of U*V was defined earlier as "**approximately equal**" and given as shown in the adjacent table.

R

X/Y	1	2	3
1	1.0	0.8	0.4
2	0.8	1.0	0.8
3	0.4	0.8	1.0

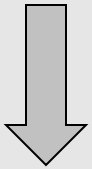
S

X/Y	1	2	3
1	0	0.6	0.8
2	0	0	0.6
3	0	0	0

Consider another relationship, S, of U*V in which x is "***considerably larger ***" than y for $\forall x \in U$ and $\forall y \in V$. The relationship S can be given as shown in the adjacent table.

Intersection Operation

X/Y	1	2	3
1	Min(1,0)	Min(0.8,0.6)	Min(0.4,0.8)
2	Min(0.8,0)	Min(1,0)	Min(0.8,0.6)
3	Min(0.4,0)	Min(0.8,0)	Min(1,0)



X/Y	1	2	3
1	0	0.6	0.4
2	0	0	0.6
3	0	0	0

Union Operation

X/Y	1	2	3
1	Max (1,0)	Max (0.8,0.6)	Max (0.4,0.8)
2	Max (0.8,0)	Max (1,0)	Max (0.8,0.6)
3	Max (0.4,0)	Max (0.8,0)	Max (1,0)

X/Y	1	2	3
1	1.0	0.8	0.8
2	0.8	1.0	0.8
3	0.4	0.8	1.0

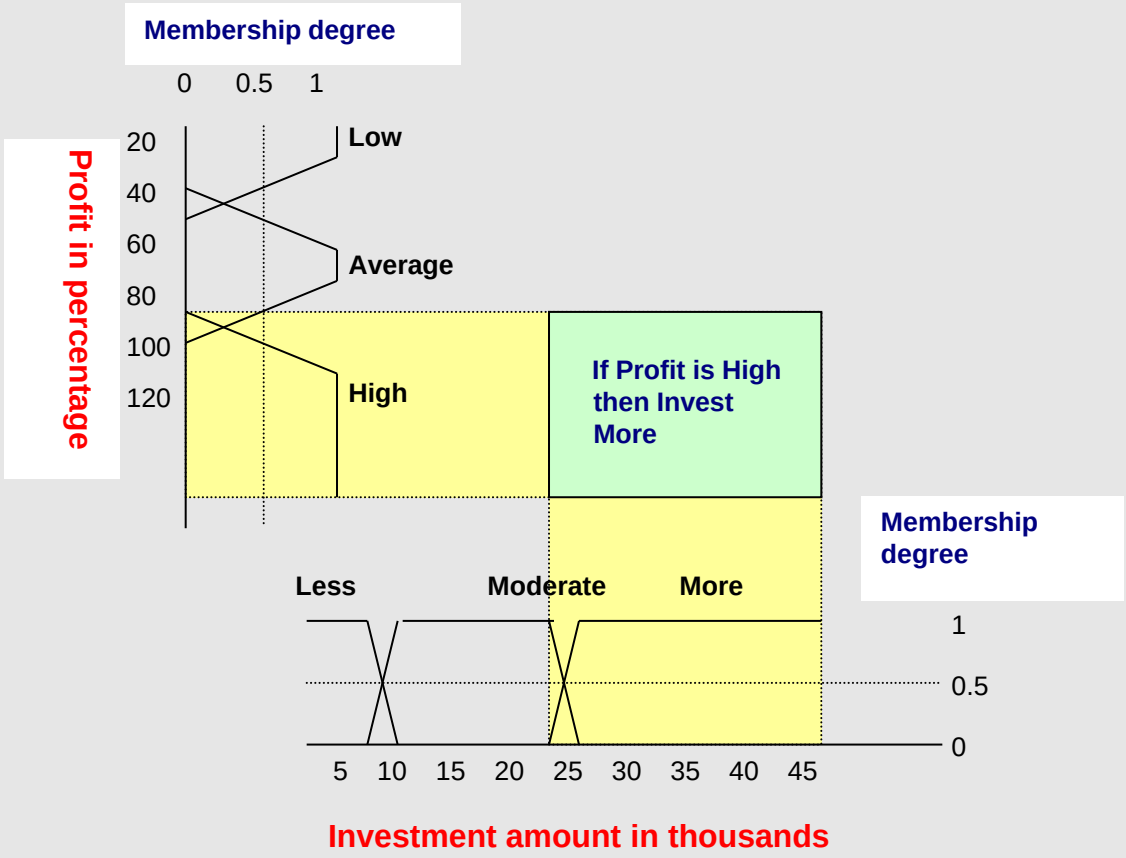
Fuzzy Connectives

- Fuzzy connectives are used to join simple fuzzy propositions to make compound propositions.

- Negations (~), disjunctions ($\cup$), conjunctions ($\cap$), and implications ($\rightarrow$) are used as fuzzy connectives.

Fuzzy Rule Based Systems

Fuzzy Portfolio Management



Fuzzy rules and relationships

Fuzzy Rule Based System

Fuzzy Washing Machine

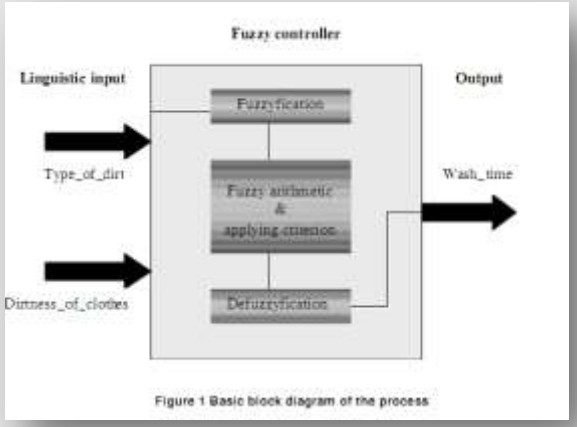
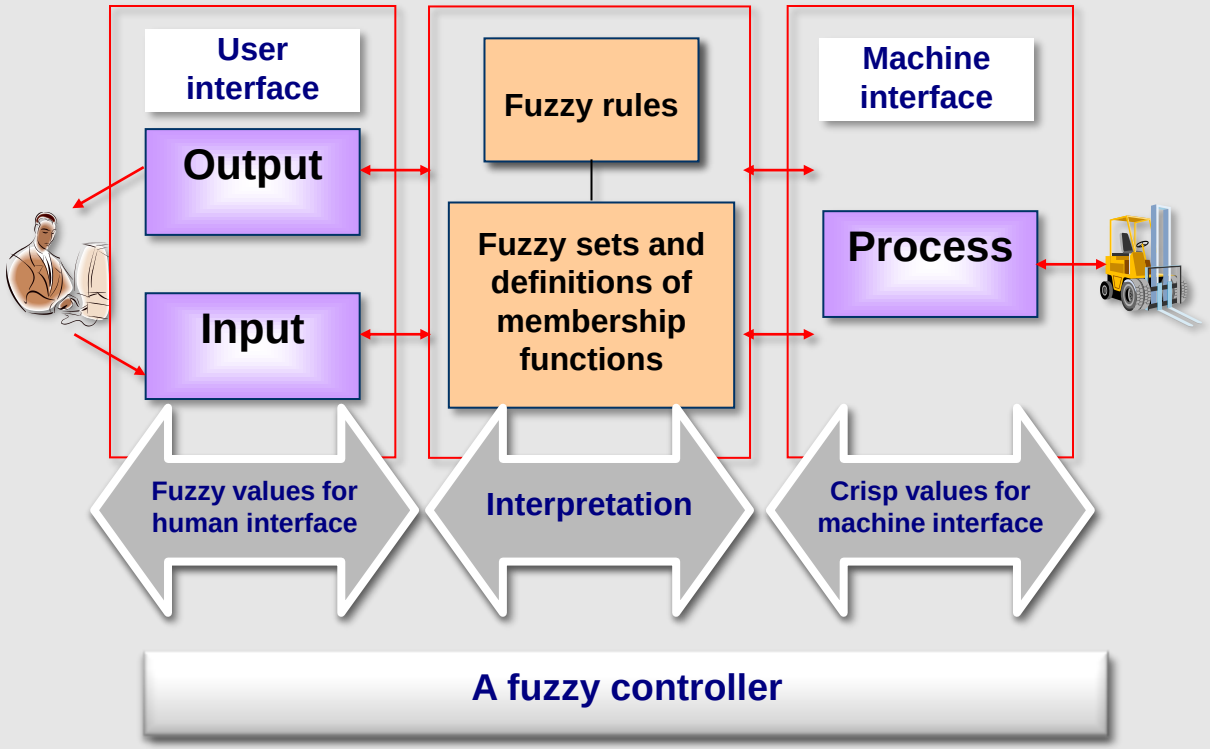


Figure 1 Basic block diagram of the process

If dirtness_of_clothes is Large and type_of_dirt is Greasy then wash_time is VeryLong;
If dirtness_of_clothes is Medium and type_of_dirt is Greasy then wash_time is Long;
If dirtness_of_clothes is Small and type_of_dirt is Greasy then wash_time is Long;
If dirtness_of_clothes is Large and type_of_dirt is Medium then wash_time is Long;
If dirtness_of_clothes is Medium and type_of_dirt is Medium then wash_time is Medium;
If dirtness_of_clothes is Small and type_of_dirt is Medium then wash_time is Medium;
If dirtness_of_clothes is Large and type_of_dirt is NotGreasy then wash_time is Medium;
If dirtness_of_clothes is Medium and type_of_dirt is NotGreasy then wash_time is Short;
If dirtness_of_clothes is Small and type_of_dirt is NotGreasy then wash_time is VeryShort.

Fuzzy rule Based System: General Structure



Fuzzy Rule Based System: An Exmple

Tip to the Waiter

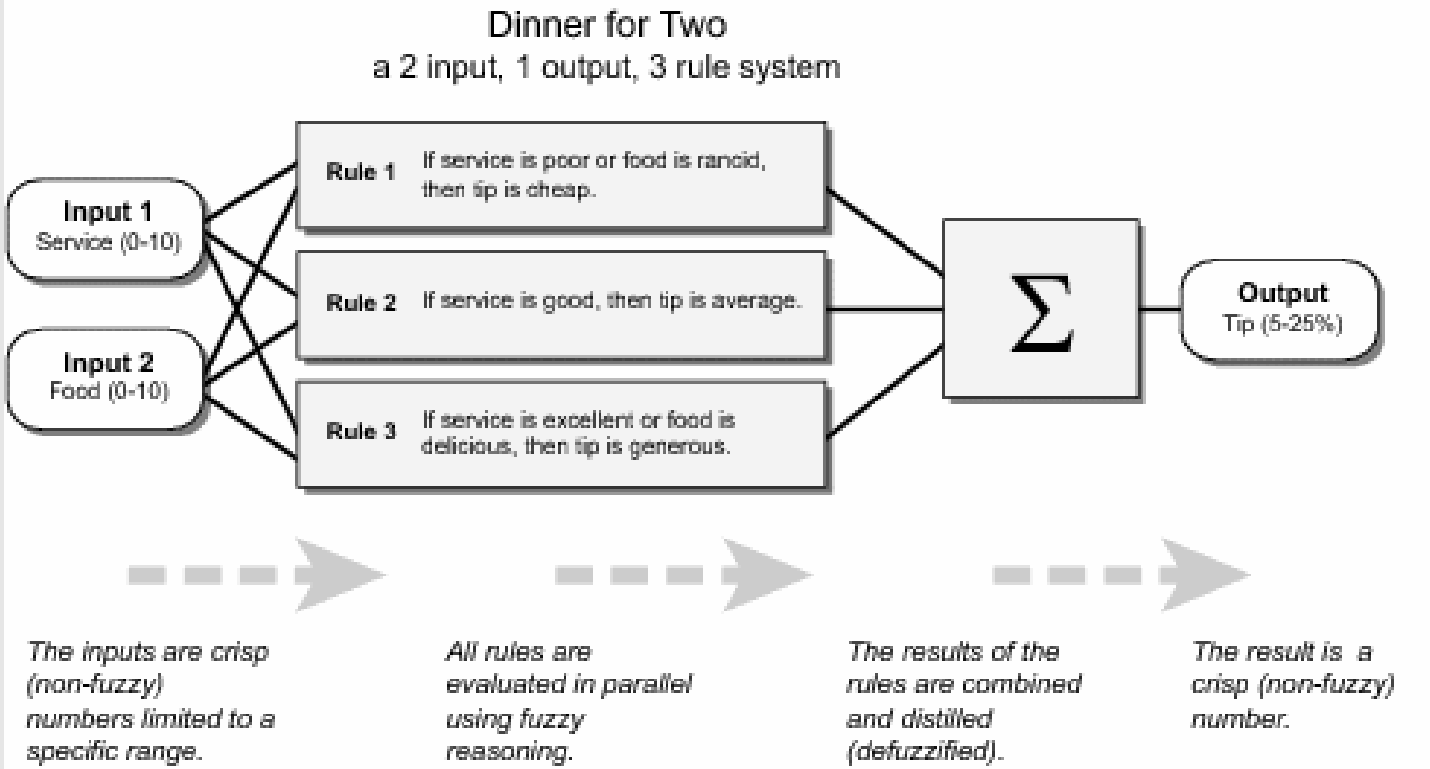
- Tip given to a waiter after a meal can be dependent of factors such as:
  - Good quality and taste of food and
  - Efficient service.

  - It should be noted that
    - **if food is delicious and service is excellent than tip is generous.**

  - The example involves determination of fuzzy mebership functions for
    - **Taste of food**
    - **Type of service**
    - **Amount of tip given to waiter**

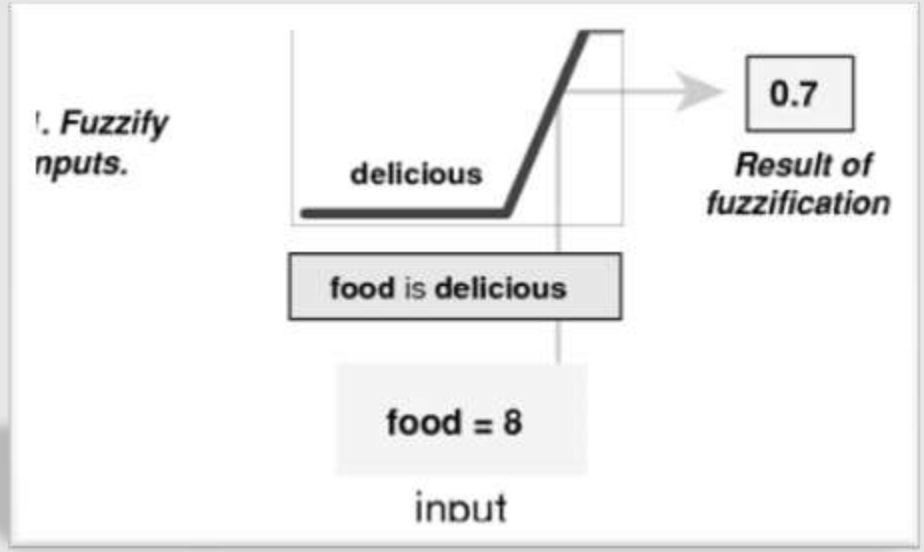
Fuzzy Rule Based System: An Exmple

Tip to the Waiter



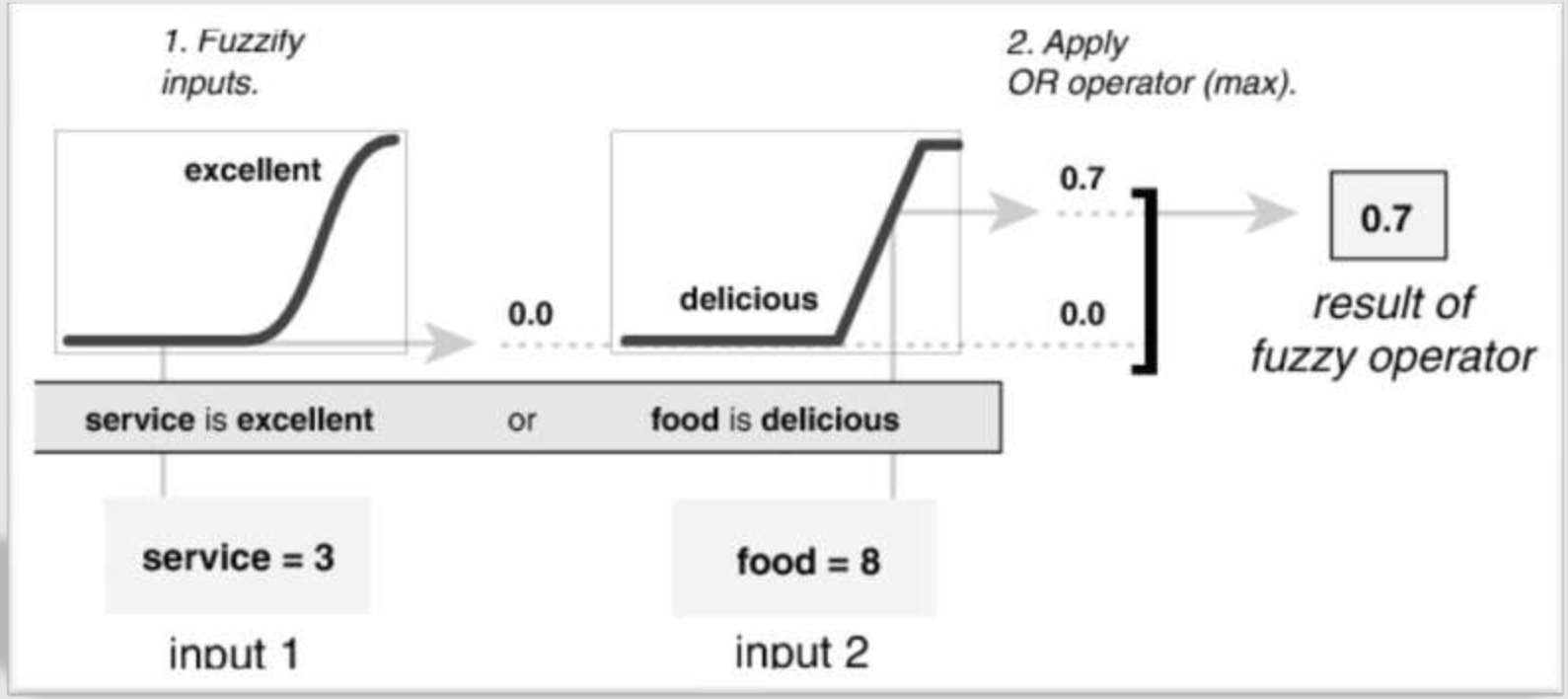
Fuzzy Rule Based System: An Exmple

Food is delicious ....



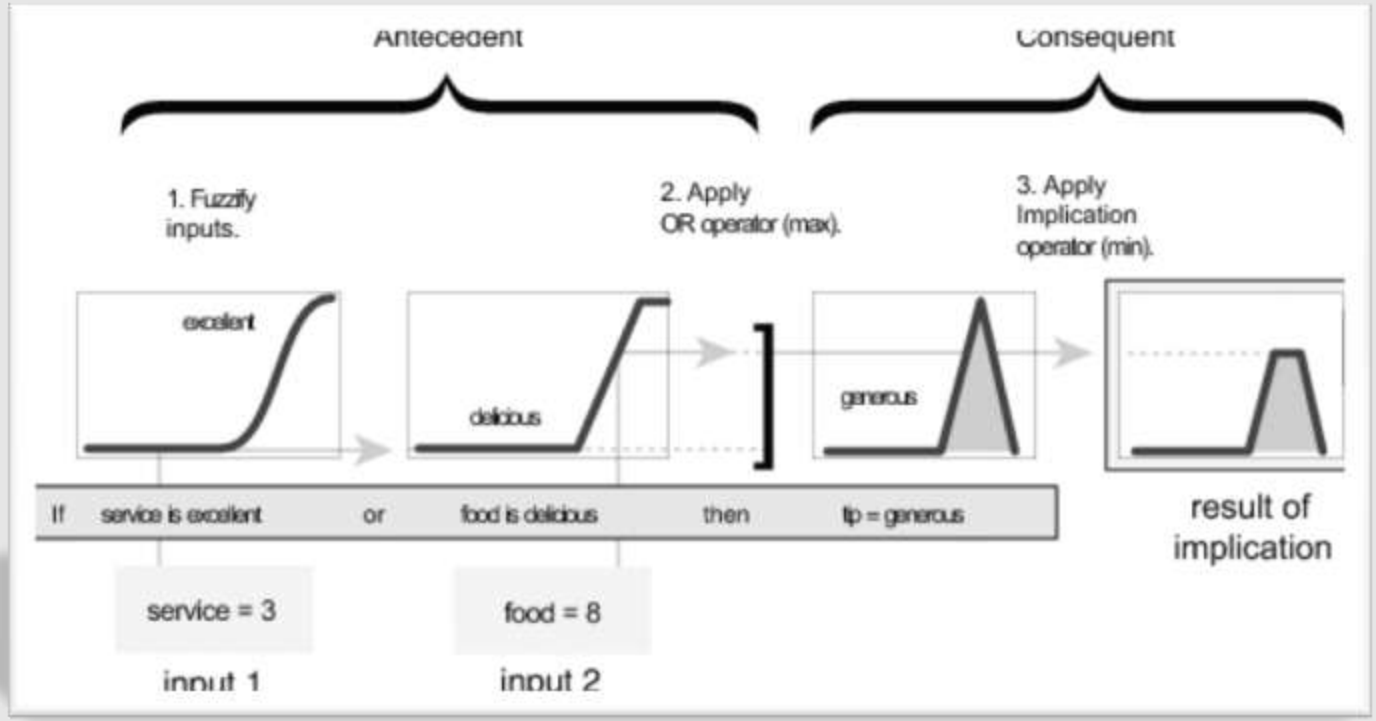
Fuzzy Rule Based System:An Exmple

Food is delicious ....

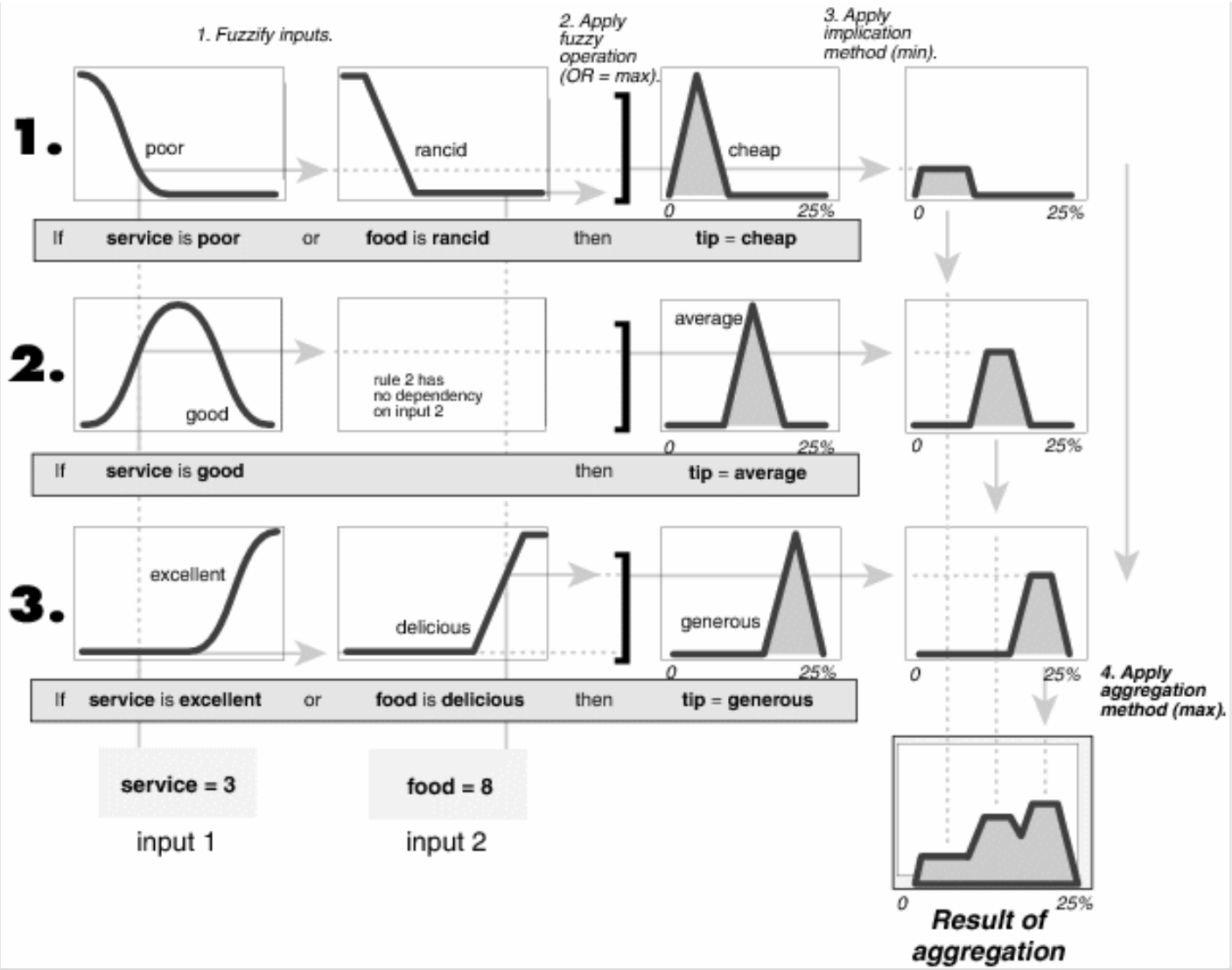


Fuzzy Rule Based System:An Exmple

Food is delicious ....



Fuzzy Rule Based System :An Example

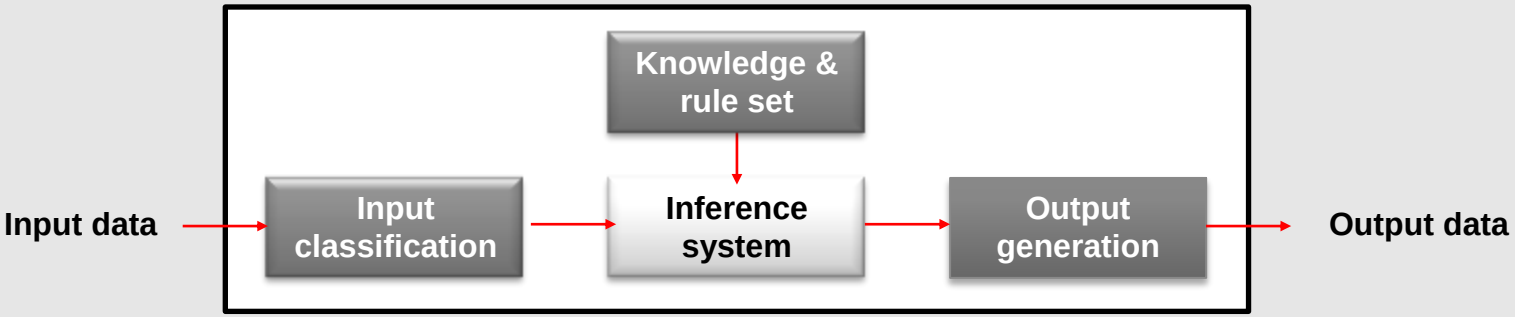


Fuzzy Logic: Advantages

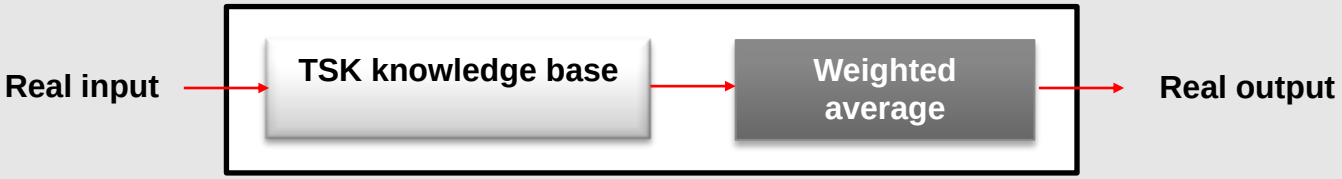
Advantages of Fuzzy Logic

- Linguistic values used make it more **human oriented.**
- Allows the **solution to previously unsolved problems.**
- **Rapid prototyping** is possible as knowledge is not required before starting work.
- **Cheaper** to make than conventional system as **easier to design**
- Increased robustness.
- **Simpler knowledge** acquisition and representation.
- A **few rules** are used to describe **great complexity.**

Models of Rule-Based Systems



(a) Mamdani model of FRBS



(b) TSK model of FRBS

General model of FRBS

This slide show is available at Pritisajja.info

Thanks!

References:

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